



ELSEVIER

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)



Economics Letters 81 (2003) 373–378

**economics  
letters**

[www.elsevier.com/locate/econbase](http://www.elsevier.com/locate/econbase)

## A three-regime real-time indicator for the US economy

Laurent Ferrara\*

*Centre d'Observation Economique, 27, Avenue de Friedland, 75382 Paris Cedex 08, France*

Received 10 February 2003; received in revised form 11 June 2003; accepted 24 June 2003

---

### Abstract

This note proposes a monthly real-time cyclical indicator for the US economy computed through a three-regime multivariate Markov-Switching model. The model specification is based on the reconciliation of the concepts of growth and business cycles.

© 2003 Elsevier B.V. All rights reserved.

*Keywords:* Multivariate Markov-Switching; Specification; Economic cycles; Real-time indicator

*JEL classification:* C32; C51; E32

---

### 1. Introduction

Recently, we witnessed the explosion of econometric papers dealing with nonlinear modeling of macroeconomic time series with the aim to describe the economic fluctuations. Indeed, it turns out that this kind of model is well designed to take into account certain stylized facts of the cycle such as asymmetries in the phases. Especially, Markov-Switching (MS hereafter) models popularized by Hamilton (1989) have been extensively used in practice, providing meaningful results on turning points detection. However, the specification of this model is not an easy task. In this note, we propose a three-regime specification procedure of the MS model for macroeconomic time series, based on the reconciliation of the concepts of economic growth and business cycles. As an application, we develop a multivariate model to build a monthly real-time cyclical indicator for the US economy.

---

\* Tel.: +33-1-55-65-70-92; fax: +33-1-55-65-70-74.

*E-mail address:* [lferrara@ccip.fr](mailto:lferrara@ccip.fr) (L. Ferrara).

*URL:* <http://www.coe.ccip.fr>.

## 2. The MS-VAR model

The MS model has been popularized in economics by Hamilton (1989) in order to take into account a certain type of nonstationarity inherent to some macroeconomic or financial time series, such as frequent shifts in mean, that cannot be caught by classical linear models. The observed time series is assumed to be approximated by an autoregressive process, whose parameters evolve through time. Moreover, their evolution is ruled by an unobservable variable which in turn follows a first-order  $K$ -state Markov chain, independent of past observations on the observed series. Krolzig (1997) proposes a multivariate extension of this model by developing a MS-VAR process whose definition is recalled below.

The  $N$ -dimensional process  $(Y_t)_t$  is said to be a MS(K)-VAR( $p$ ) process if it verifies the following equation:

$$Y_t - \mu(S_t) = \sum_{j=1}^p \phi_j(S_t)(Y_{t-j} - \mu(S_{t-j})) + \varepsilon_t \quad (1)$$

where  $(\varepsilon_t)_t$  is a Gaussian white noise process with finite variance–covariance matrix  $\Sigma(S_t)$ , where  $\phi_j$  is an  $N \times N$  matrix, for  $j=1, \dots, p$ , and where the unobservable discrete variable  $(S_t)_t$  is supposed to represent the current state of the economy (for all  $t$ ,  $S_t \in \{1, \dots, K\}$ ,  $K$  being the number of regimes). This previous form is known as the mean-adjusted form of the MS(K)-VAR( $p$ ) model. Moreover, the whole specification of the model needs the specification of  $(S_t)_t$  as a  $K$ -state first-order Markov chain. That is, the value of the time series  $S_t$  for all  $t$  depends only on the last value  $S_{t-1}$ , i.e., for  $i, j=1, \dots, K$ :

$$P(S_t = j \mid S_{t-1} = i, S_{t-2} = i, \dots) = P(S_t = j \mid S_{t-1} = i) = p_{ij}. \quad (2)$$

The probabilities  $(p_{ij})_{i,j=1, \dots, K}$  are called *transition probabilities* of moving from one state to the other.

Regarding the parameter estimation issue from an observed  $N$ -dimensional trajectory  $(y_1, \dots, y_T)$ , the maximum likelihood method is used in connection with the expectation-maximisation (EM) algorithm, which has been proved to be more robust to the starting parameters values. As a by-product of the estimation step, we get the estimated conditional probabilities of being in the state  $i$ , for  $i=1, \dots, K$ ,  $P(S_t = i \mid y_{t-1}, \dots, y_1, \theta)$ , referred to as the *filtered probability*, and  $P(S_t = i \mid y_T, \dots, y_1, \theta)$ , referred to as the *smoothed probability*. Both probabilities are used, respectively, to detect in real time and to date ex post the turning points of the economic cycles.

## 3. Specification in business cycles analysis

The main issue in the MS specification relates to the number of regimes. Unfortunately, as noted by Hamilton, the assumption that the process describing the data presents a given number  $K$  of regimes cannot be tested using the usual likelihood ratio test, because some of the classical regularity conditions are not fulfilled, although some specification tests have been proposed in the literature (see for instance Hansen, 1992, 1996; Hamilton, 1996; Garcia, 1998). In business cycle analysis, a second validation

stage is needed to test the ability of the model to replicate some business cycle characteristics, for instance, by using numeric simulations (Clements and Krolzig, 2000; Breunig and Pagan, 2001). Indeed, the regime estimates do not have spontaneously a clear interpretation, and a priori, there is no reason why the estimated regimes would coincide with the economic cycles phases.

Generally, in business cycle analysis, a two-state Markov chain is used in applications in order to match with the seminal characterization of the cycle in terms of expansion and contraction given by Burns and Mitchell (1946). However, since the paper of Sichel (1994), some authors found evidence in favor of a three-regime model for the business cycle (Krolzig and Toro, 1999; Layton and Smith, 2000). Either the expansion phase can be separated into a regular-growth phase and a high-growth phase, or the contraction phase can also be separated into a slowdown phase and a recession phase. In some empirical studies, the third regime may also be used as a dummy variable to deal with the nonstationarity in mean of the series.

Instead of a data-driven choice of  $K$ , we propose to impose a priori a three-regime specification based on an economic approach in terms of cycles. Two type of cycles are generally considered separately in economics: the business cycle and the growth cycle. First, the business cycle of a country is meant to reproduce the cycle of the global level of activity. It is the most common definition in the economics literature, introduced by the NBER. The turning points of that cycle separate periods of negative growth (recessions) from periods of positive growth (expansions). In practice, a period of negative growth will be admitted as a recession if it respects minimum criteria relative to duration and intensity as well as a sufficient diffusion in the economy. The second cycle, largely discussed in Europe, is the growth cycle, introduced by the OECD in the 1960s. This cycle can be defined as the deviation of the reference series (GDP for example) to its secular trend, though difficult to estimate through filters (e.g., Hodrick–Prescott, Baxter–King, . . .). The growth cycle turning points have a clear meaning. The peak is reached when the growth rate decreases below the trend growth rate, and symmetrically, the trough is reached when the growth rate overpasses it again. In other words, the peak is reached when the first derivative of the deviation to trend becomes null, i.e., when the derivative of the series (representing the instantaneous growth rate) equals the derivative of the trend. The two phases of the growth cycle are referred to as slowdown and rebound phases.

Regarding the connections between both cycles, if the slowdown gains in intensity, the growth rate will decrease below the trend growth rate, and finally, if it is really getting worse, the growth rate will become negative, provoking a recession. We get a nonsymmetric chronology for upward phases: first, exit of recession and, second, of the growth cycle. However, a slowdown will not translate automatically into a recession (see, for instance, Anas and Ferrara, 2002b, for more detailed connections between both cycles). According to this approach of the cycles, we consider a three-regime MS-VAR model with the following economic interpretation:

- low regime ( $S_t=1$ ): the economy is in recession (low phase of the business cycle)
- intermediate regime ( $S_t=2$ ): the growth rate of the economy is below its trend growth rate (low phase of the growth cycle without recession)
- high regime ( $S_t=3$ ): the growth rate of the economy is over its trend growth rate (high phase of the growth cycle)

It is noteworthy that this latter interpretation of the three regimes assumes a constant long-term growth rate over the whole sample period.

#### 4. An application to the US economy

In this section, we develop a three-regime MS-VAR model for the American economy, from January 1985 to November 2002, to build a real-time regime indicator. The four selected coincident variables are the inverted unemployment rate of civilian workers, manufacturing industrial production index, help-wanted advertising index released by the Conference Board, and construction spending for the private sector (see [Anas and Ferrara, 2002a](#), as regards the choice of these series). To ensure stationarity and to exclude the very short-term fluctuations, we consider the 3-month growth rates of the series (5 months for the unemployment). The model is specified as having the same variance for each regime and no autoregressive lag. Indeed, the presence of lags and heteroscedasticity deteriorates the reproduction of the stylized facts of the cycles. Parameters are estimated by the EM algorithm, through the MSVAR module of the Ox package (see [Doornik, 1999](#)) developed by Krolzig, and are presented in [Table 1](#). Note that this study is not based on historically released data which are often difficult to get (see the recent work carried out by [Chauvet and Piger, 2003](#), on US GDP and employment).

The results in [Table 2](#) show that persistence is quite high since the probability to stay in the same regime exceeds 88% in the three regimes. The transition probability from a high regime to a low regime is null, which means that before entering into recession, the economy always first decelerates. Moreover, the transition probability from a low regime to a high regime is quasi-null, which means that the two last recessions have been followed by a slow recovery. The average duration of recessions is around 8 months, 12 months for intermediate regime and 27 months for high regime. This latter high value is due to the influence of the new economy on the productivity during the second part of the 1990s in the United States.

In order to provide a real-time regime indicator, we get the three filtered probabilities of being in each regime, denoted  $P(S_t = i | y_1, \dots, y_t)$ , for  $i = 1, 2$ , and 3, to classify the observations between regimes (see [Fig. 1](#)). Regarding the decision rule of the indicator, we assign the observation at time  $t$  to the regime with the higher probability, i.e.:

$$i^* = \text{Arg max} P(S_t = i | y_1, \dots, y_t).$$

The indicator shows that the United States has experienced slowdowns without recession in 1985–1986 and 1995 and two recessions in 1990–1991 and 2001. The results underline a pretty good fit of the estimated recession periods with the official dates released by the NBER. Moreover, the estimated dates of the peaks and troughs of the growth cycle match with the dates provided by the [OECD \(2002\)](#), though the growth cycle is not estimated by the same method. Specifically, as concerns the last cycle, the results

Table 1  
Estimated parameters of the MS(3)-VAR(0) model

	Unemployment	IPI	Help-wanted index	Construction
$\mu(S_t = 1)$	– 0.1281	– 0.0163	– 0.1304	– 0.0297
$\mu(S_t = 2)$	– 0.0206	0.0033	– 0.0201	– 0.0027
$\mu(S_t = 3)$	0.0413	0.0136	0.0123	0.0093
Standard error	0.0339	0.0078	0.0377	0.0213

Table 2

Estimated transition probabilities of the MS(3)-VAR(0) model

	$p_{i1}$	$p_{i2}$	$p_{i3}$	Duration	Unconditional probability
Regime 1	0.8820	0.1174	0.0006	8.48	0.0804
Regime 2	0.0265	0.9152	0.0583	11.80	0.3581
Regime 3	0.0000	0.0372	0.9628	26.85	0.5615

show that the slowdown began in August 2000 and translated into a recession 7 months later, in March 2001. This recession was pretty mild and lasted 9 months, until December 2001. Since then, the US economy stays in the intermediate regime: In November 2002, the probability of being in the intermediate regime was 99.8%. That is, the growth rate of the economy is currently below its trend growth rate estimated around 3% per year over the considered period (1985 Q1–2002 Q4). Indeed, this result is confirmed by the annual growth rate in 2002 estimated in January 2003 around 2.4%. This is mainly due to the high unemployment rate (6% in November 2002) which curbs the consumption spending and to the prospect of war in Iraq which does not encourage companies to invest. This coincident regime indicator can be computed each month to detect in real time the regimes of the US economy.

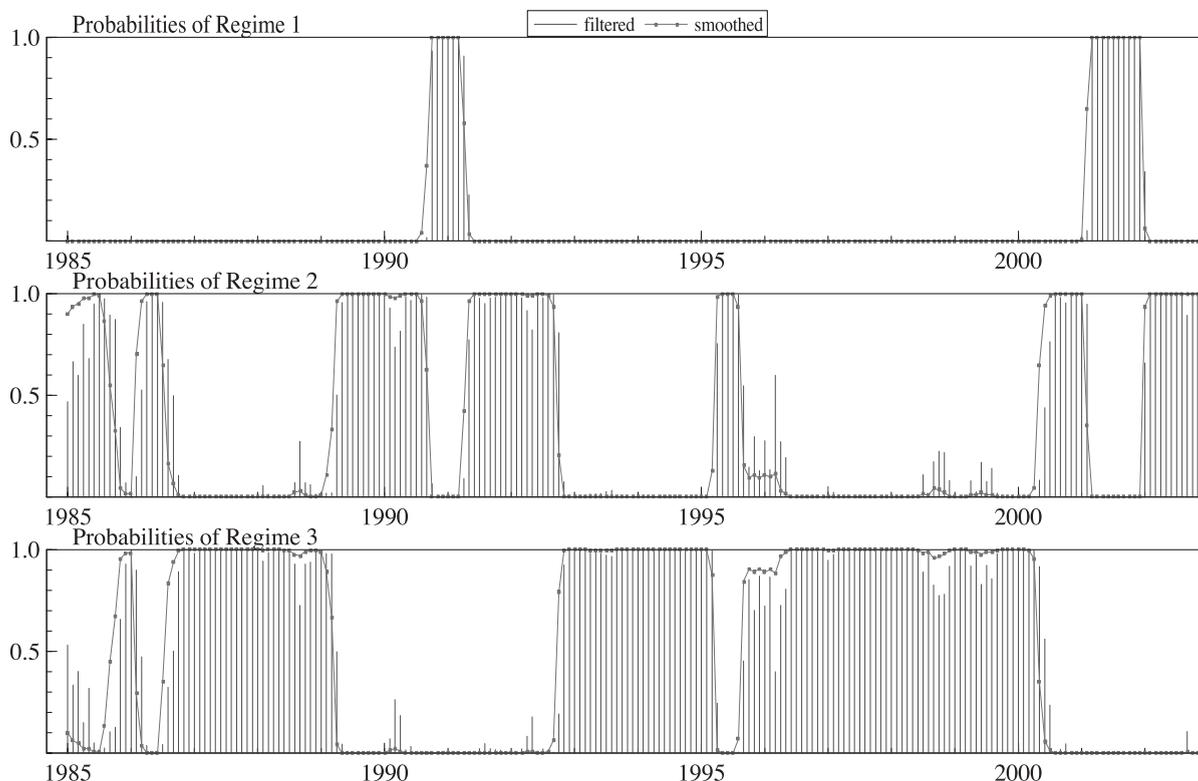


Fig. 1. Filtered and smoothed probabilities of being in a given regime provided by the MS(3)-VAR(0) model.

## Acknowledgements

The author would like to thank J. Anas and an anonymous referee for their helpful comments.

## References

- Anas, J., Ferrara, L., 2002a. Un indicateur d'entrée et sortie de récession: application aux Etats-Unis, Document de travail No. 58, Centre d'Observation Economique ([www.coe.ccip.fr](http://www.coe.ccip.fr)).
- Anas, J., Ferrara, L., 2002b. Detecting cyclical turning points: the ABCD approach and two probabilistic indicators. Paper Presented at the 26th Cirt Conference in Taiwan, October 2002 ([www.coe.ccip.fr](http://www.coe.ccip.fr)). forthcoming in *Journal of Business Cycle Measurement and Analysis*.
- Breunig, R., Pagan, A., 2001. Some simple methods for assessing Markov-Switching models, mimeo. Australian National University.
- Burns A.F., Mitchell, W.C., 1946. *Measuring Business Cycles* (NBER, Columbia University Press).
- Chauvet, M., Piger, J.M., 2003. Identifying business cycle turning points in real time. Review of the Federal Reserve Bank of St. Louis. pp 47–61. March–April.
- Clements, M.P., Krolzig, H.M., 2000. Modelling business cycle features using switching regimes models, mimeo.
- Doornik, J.A., 1999. *Object-Oriented Matrix Programming using Ox*, 3rd ed. Timberlake Consultant Press, London.
- Garcia, R., 1998. Asymptotic null distribution of the likelihood ratio test in Markov-Switching models. *International Economic Review* 39, 763–788.
- Hamilton, J.D., 1989. A new approach to the economic analysis of non-stationary time series and the business cycle. *Econometrica* 57 (2), 357–384.
- Hamilton, J.D., 1996. Specification testing in Markov-switching time series models. *Journal of Econometrics* 70, 127–157.
- Hansen, B.E., 1992. The likelihood ratio test under non-standard conditions: testing the Markov trend model of GNP. *Journal of Applied Econometrics* 7, S61–S82 (supplement).
- Hansen, B.E., 1996. Erratum: the likelihood ratio test under non-standard conditions: testing the Markov trend model of GNP. *Journal of Applied Econometrics* 11, 195–198.
- Krolzig, H.M., 1997. *Markov-Switching Vector Autoregressions. Modelling, Statistical Inference and Applications to Business Cycle Analysis*. Springer, Berlin.
- Krolzig, H.M., Toro, J., 1999. A new approach to the analysis of shocks and the cycle in a model of output and employment. European University Institute. Working paper ECO, no. 99-30.
- Layton, A.P., Smith, D., 2000. Further on the three phases of the US business cycle. *Applied Economics* 32, 1133–1143.
- OECD, 2002. *An update of the OECD Composite Leading Indicators* (Statistics Directorate, OECD, December 2002).
- Sichel, D.E., 1994. Inventories and the three phases of the business cycle. *Journal of Business and Economic Statistics* 12, 269–277.